A Century of Interfuel Substitution*

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Abstract:

We investigate the demand for energy and the degree of substitutability among fossil fuels in the United States using the Normalized Quadratic (NQ) expenditure function and (to our knowledge) the longest span prices and quantities that have ever been studied before, from 1919 to 2012. In doing so, we merge the empirical energy demand systems literature with the recent financial econometrics literature, relaxing the homoskedasticity assumption and instead assuming that the covariance matrix of the errors of the flexible demand system is time-varying. We generate inference, in terms of a full set of elasticities, consistent with neoclassical microeconomic theory and the data generating process. Our results are in line with earlier findings in the literature based on shorter duration samples and different methodologies. They have important implications for climate policy intervention designed to reduce greenhouse gas emissions. We show that there is a small but statistically significant substitution possibility between crude oil and natural gas, but not between crude oil and coal. We also provide evidence that natural gas is a substitute for coal when the price of coal changes, but coal is not a substitute for natural gas when the price of natural gas changes.

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1 Introduction

In recent years, the main approach to the investigation of interfuel substitution (energy elasticities) and the demand for energy is based on the dual approach to demand system generation developed by Diewert (1974). It allows estimation in a systems context, assuming a flexible functional form for the aggregator function, and the computation of the relevant elasticity measures like the income elasticities, the own- and cross-price elasticities, and the Allen and Morishima elasticities of substitution. It also allows us to achieve theoretical regularity (in terms of curvature, positivity, and monotonicity of the aggregator function), but it is difficult to simultaneously achieve econometric regularity (in terms of stationary equation errors), because the combination of nonstationary data and nonlinear estimation of demand systems is a difficult issue and has not yet been addressed in the literature.

The flexible functional forms approach was pioneered by Berndt and Wood (1975), Fuss (1977), and Pindyck (1979) in the context of interfactor and interfuel substitution. Thanks to the growing interest, there has been a large number of studies investigating interfuel substitution and the demand for energy employing flexible functional forms — see, for example, Uri (1979), Considine (1989), Hall (1986), and Jones (1995), among others. However, the major contributions in this area use data before the 1970s and are based on flexible functional forms with small theoretical regularity regions. Also, as noted by Serletis *et al.* (2010) and Jadidzadeh and Serletis (2016) most of these studies ignore the theoretical regularity conditions. Moreover, all of the studies in the interfuel substitution literature assume homoskedastic errors despite recent leading-edge research in the financial econometrics literature that has introduced new models in which stochastic variables are assumed to have a time-dependent variance.

In this article, we investigate interfuel substitution in the United States in the context of the locally flexible Normalized Quadratic (NQ) expenditure function, developed by Diewert and Wales (1988), as in Serletis *et al.* (2010) and Jadidzadeh and Serletis (2016). In doing so, however, we follow Serletis and Isakin (2017) and Serletis and Xu (2017) and introduce recent state-of-the-art advances in financial econometrics to the empirical energy demand literature. In particular, we relax the homoskedasticity assumption and instead assume that the covariance matrix of the errors of the demand system is time varying, thus improving the flexibility of the demand system to capture important features of the data. We also pay explicit attention to theoretical regularity, as suggested by Barnett (2002).

To our knowledge, we use the longest span prices and quantities that have ever been studied before, over nearly 100 years (from 1919 to 2012), to investigate the substitutability/complementarity relationship between crude oil, coal, and natural gas. Although crude oil is not consumed directly, but is used as a factor of production in the refining industry (in the production of gasoline, diesel, heating oil, and jet fuel), it competes with natural gas, which also competes with coal in producing electricity and in the manufacturing of chemicals and metals. Based on the Normalized Quadradic expenditure function and by relaxing the assumption of homoskedasticity (imposing a GARCH-BEKK (1,1) specification to the conditional variance matrix), we estimate a set of income elasticities, own- and cross-price elasticities, and Allen and Morishima elasticities of substitution, consistent with neoclassical microeconomic theory and the data generating process. We find that there is a small but statistically significant substitution possibility between crude oil and natural gas, and that natural gas is a substitute for coal when the price of coal changes, but coal is not a substitute for natural gas when the price of natural gas changes. We also estimate the model over two subsamples, from 1919 to 1972 and from 1973 to 2012, to account for a structural break in the long series and enhance the robustness of our results.

The rest of the paper is organized as follows. Section 2 provides a brief literature review and Section 3 sketches out related neoclassical microeconomic theory and applied consumption analysis. Section 4 discusses related econometric issues, paying explicit attention to the singularity problem, the imposition of global concavity, and the modelling of the conditional covariance matrix of the NQ demand system. Section 5 discusses the data and section 6 presents the NQ model with heteroskedastic disturbances. Sections 7 and 8 present the empirical results, section 9 discusses the policy implications of our findings, and section 10 compares the reported results to those in the literature. The final section concludes the paper.

2 Literature Review

As noted in the Introduction, influenced by the pioneering work by Berndt and Wood (1975), Fuss (1977), and Pindyck (1979) in the context of interfactor and interfuel substitution, a large number of early studies in the energy demand literature took the flexible functional forms approach. See Table 1, which is reproduced (and expanded) from Jadidzadeh and Serletis (2016), for a list of some of these studies. Most of the early studies, however, focused mainly on the role of energy in the structure of production and investigated interfactor substitutability — see, for example, Berndt and Wood (1975) and Fuss (1977). Over the years, a number of studies also focused on the consumption side and investigated the demand for energy and interfuel substitution — see, for example, Hall (1986), Considine (1989), and Jones (1995), among others.

As can be seen in Table 1, an important feature of the early literature is the use of locally flexible functional forms, in particular the translog, introduced by Christensen *et al.* (1975). However, as argued by Caves and Christensen (1980), Guilkey and Lovell (1980), Barnett and Lee (1985) and Barnett *et al.* (1985, 1987), the translog fails to meet the theoretical regularity conditions of neoclassical microeconomic theory (positivity, monotonicity, and curvature) in large regions. A result was the development of locally flexible functional forms that have large (but not global) regular regions, such as the quadratic AIDS (QUAIDS), developed by Banks *et al.* (1997), and the Minflex Laurent (ML) model, developed by Barnett (1983).

Cooper and McLaren (1996) classify those models as 'effectively globally regular' flexible functional forms, because they typically have regular regions that include almost all data points in the sample. In addition, Diewert and Wales (1988) proposed two locally flexible functional forms, for which the theoretical curvature conditions can be imposed globally — the normalized quadratic (NQ) reciprocal indirect utility function and the NQ expenditure function.

The most recent energy demand literature makes use of the effectively globally regular and normalized quadratic flexible functional forms. For example, Chang and Serletis (2014) use the quadratic AIDS and the Minflex Laurent model to investigate the demand for gasoline on three expenditure categories in the Canadian transportation sector. Also, as noted in Table 1, Serletis *et al.* (2010, 2011) and Jadidzadeh and Serletis (2016) use the normalized quadratic function in the interfuel substitution literature and achieve the theoretical curvature conditions globally. Moreover, Serletis and Shahmoradi (2008) use two globally flexible functional forms — the Fourier, introduced by Gallant (1981), and the Asymptotically Ideal Model (AIM), employed and explained in Barnett and Yue (1988) — to investigate interfuel substitution in energy demand in the United States. In this paper, we follow Serletis *et al.* (2010, 2011) and Jadidzadeh and Serletis (2016) and use the normalized quadratic function. In doing so, we also introduce recent advances in financial econometrics, relaxing the homoskedasticity assumption that all of the earlier papers listed in Table 1 make.

3 The Structure of Preferences

We assume that (in the general case with n energy) the group of n energy goods is homothetically weakly separable from the non-energy goods in the underlying utility function. Therefore, the utility function f is assumed to take the following form

$$U = f\left(E\left(\boldsymbol{x}\right), \boldsymbol{M}\right) \tag{1}$$

where U is gross utility, $E(\cdot)$ is a homothetic aggregator function over the n energy goods, $\boldsymbol{x} = (x_1, ..., x_n)$, and $\boldsymbol{M}$ is a vector of other non-energy goods. The weak separability condition requires that the marginal rate of substitution between any two components of $\boldsymbol{x}$ does not depend upon the value of $\boldsymbol{M}$.

Under this assumption and duality theorems [see Diewert (1974)], the corresponding expenditure function can be written as

$$C = g\left(P_E\left(\boldsymbol{p}\right), \boldsymbol{p}_M, U\right)$$

where $\boldsymbol{p} = (p_1, ..., p_n)$ is the corresponding price vector of the n energy goods, $\boldsymbol{p}_M$ that of the non-energy goods, and $P_E(\cdot)$ is an energy price aggregator function which is a homothetic function and can be represented by a unit expenditure function.

3.1 The NQ Expenditure Function

Our objective is to estimate a system of demand equations consistent with neoclassical microeconomic theory. In order to do so, we use a flexible functional form capable of approximating an arbitrary twice continuously differentiable function to the second order at an arbitrary point in the domain. Moreover, the flexible functional form that we use allows for the imposition of global curvature without losing its flexibility property. The flexible functional form that we use is the Normalized Quadratic (NQ) expenditure function, developed by Diewert and Wales (1988). For a given utility level, u, and vector of prices $\boldsymbol{p} = [p_1, p_2, \dots, p_n]^T$, the NQ expenditure function is defined as (in the general case with n goods)

$$C(\boldsymbol{p}, u) = \boldsymbol{\theta}^T \boldsymbol{p} + u\left(\boldsymbol{b}^T \boldsymbol{p}\right) + 0.5u\left(\boldsymbol{p}^T \boldsymbol{B} \boldsymbol{p} / \boldsymbol{\alpha}^T \boldsymbol{p}\right) \tag{2}$$

where $\boldsymbol{\theta} = [\theta_1, \theta_2, \dots, \theta_n]^T$, $\boldsymbol{b} = [b_1, b_2, \dots, b_n]^T$, and the elements of the $n \times n$ matrix $\boldsymbol{B} \equiv [\beta_{ij}]$ are the unknown parameters to be estimated. The non-negative vector $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \dots, \alpha_n]^T$ can be predetermined as a vector of ones ($\boldsymbol{\alpha} = \mathbf{1}_n$) — see Diewert and Fox (2009) for more details.

To ensure the flexibility and Gorman polar form of the NQ function, we follow Diewert and Wales (1988) and impose the following restrictions

$$\boldsymbol{\alpha}^T \boldsymbol{p}^* = 1, \quad \boldsymbol{\alpha} \geq \mathbf{0}_n \tag{3}$$

$$\boldsymbol{\theta}^T \boldsymbol{p}^* = 0 \tag{4}$$

and

$$\boldsymbol{B} \boldsymbol{p}^* = \mathbf{0}_n, \; \boldsymbol{B} = \boldsymbol{B}^T \tag{5}$$

where $\boldsymbol{p}^* \gg \mathbf{0}_n$ is a reference (or base-period) vector of normalized prices, determined in such a way that $\boldsymbol{p}^* = \mathbf{1}_n$.

The NQ demand system in budget share form is

$$\boldsymbol{s}(\boldsymbol{v}) = \widehat{\boldsymbol{v}} \boldsymbol{\theta} + \widehat{\boldsymbol{v}} \frac{\boldsymbol{b} \boldsymbol{\alpha}^T \boldsymbol{v} + \boldsymbol{B} \boldsymbol{v} - 0.5(\boldsymbol{\alpha}^T \boldsymbol{v})^{-1} \boldsymbol{v}^T \boldsymbol{B} \boldsymbol{v} \boldsymbol{\alpha}}{\boldsymbol{b}^T \boldsymbol{v} \boldsymbol{\alpha}^T \boldsymbol{v} + 0.5 \boldsymbol{v}^T \boldsymbol{B} \boldsymbol{v}} \times \left(1 - \boldsymbol{\theta}^T \boldsymbol{v}\right) \tag{6}$$

where $\boldsymbol{s}(\boldsymbol{v}) = [s_1(v), \dots, s_n(v)]^T$ with $s_i(v) = p_i x_i / y$ being the share of the ith good in total expenditure on energy consumption (y), and $\boldsymbol{v} = [v_1, \dots, v_n]^T$ is the income normalized price vector, with the jth element $v_j = p_j / y$. Further, $\widehat{\boldsymbol{v}}$ is the $n \times n$ diagonal matrix with normalized prices on the main diagonal, and $\mathbf{1}_n = [1, \dots, 1]^T$ is a vector of ones.

4 The Econometric Approach

To estimate the share equation system (6), we add a stochastic component and write it as

$$\boldsymbol{s}_t = \boldsymbol{\psi}(\boldsymbol{v}_t, \boldsymbol{\vartheta}) + \boldsymbol{\varepsilon}_t \tag{7}$$

where $\boldsymbol{\vartheta}$ is the vector of parameters to be estimated, $\boldsymbol{\varepsilon}_t$ is a vector of stochastic errors, and $\boldsymbol{\psi}(\boldsymbol{v}_t, \boldsymbol{\vartheta}) = (\psi_1(\boldsymbol{v}_t, \boldsymbol{\vartheta}), ..., \psi_n(\boldsymbol{v}_t, \boldsymbol{\vartheta}))'$, with $\psi_i(\boldsymbol{v}_t, \boldsymbol{\vartheta})$ given by the right-hand side of (6).

4.1 Estimation with Homoskedastic Disturbances

When estimating $\boldsymbol{\vartheta}$ in equation (7), a typical assumption about $\boldsymbol{\varepsilon}_t$ in the literature so far is homoskedasticity. This assumption requires

$$\boldsymbol{\varepsilon}_t \sim N(\mathbf{0}, \boldsymbol{H}) \tag{8}$$

where $\mathbf{0}$ is an n-dimensional null vector and $\boldsymbol{H}$ is the $n \times n$ symmetric positive definite error covariance matrix. The assumption of a classical disturbance term permits correlation among the disturbances at time t, but rules out the possibility of autocorrelated disturbances.

To impose restrictions (3) empirically, we set the prices to unity in the reference (base) year. Thus, equation (5) becomes

$$\sum_{j=1}^{n} \beta_{ij} = 0, \quad \forall i \quad \text{and} \quad \beta_{ij} = \beta_{ji}, \quad \forall i, j \tag{9}$$

restriction (3) reduces to

$$\alpha_i = 1, \quad \forall i$$

and restriction(4) reduces to

$$\sum_{i=1}^{n} \theta_i = 0.$$

Finally, since the demand equations, and consequently share equations (6), are homogeneous of degree zero, we follow Diewert and Wales (1988) and impose the following restriction

$$\sum_{i=1}^{n} b_i = 1.$$

Under these restrictions, the NQ expenditure system is well defined and has $(n^2 + 3n - 4)/2$ free parameters (that is, parameters estimated directly). The remaining parameters can be recovered from the restrictions imposed.

4.2 Invariance

The assumption of a classical disturbance term and the fact that $\boldsymbol{s}_t$ (and therefore the $\boldsymbol{\varepsilon}_t$) satisfy the adding up condition (i.e., the budget shares sum to 1) imply that the disturbance covariance matrix, $\boldsymbol{H}$, is singular. This introduces a technical problem when the demand system is estimated, since either generalized least squares or maximum likelihood (ML) needs to invert the covariance matrix, $\boldsymbol{H}$. Barten (1969) and McLaren (1990) show that maximum likelihood estimates can be obtained by arbitrarily dropping any equation in the system. McLaren (1990) also establishes invariance by virtue of observational equivalence of the subsystems with different deleted equations.

4.3 Econometric Regularity

In the literature, attempts have been made to achieve econometric regularity, correcting for serial correlation by allowing the possibility of a first-order autoregressive process in the error terms of equation (6). However, as also noted by Jadidzadeh and Serletis (2016), correction for serial correlation increases the number of curvature violations and also leads to induced violations of monotonicity and positivity. Moreover, it should be noted that allowing for first order serial correlation is almost the same as taking first differences of the data if the autocorrelation coefficient is close to unity. In that case, although the equation errors become stationary, there is no theory for the model in first differences. In addition, even if the errors are stationary and the estimates are super consistent, standard estimation procedures are inadequate for obtaining correctly estimated standard errors for coefficients in cointegrating equations — see, for example, Attfield (1997) and Ng (1995).

It is for these reasons that Lewbel and Ng (2005) correctly argue that the majority of the existing empirical demand system studies have failed to deal with the combination of non-stationary data and nonlinear estimation. In fact, simultaneously achieving both economic regularity and econometric regularity in the estimation of nonlinear demand systems, like the NQ demand system used in this paper, has been challenging and noted as an area for potentially productive future research by Feng and Serletis (2008).

4.4 Estimation with Heteroskedastic Disturbances

Recently, Serletis and Isakin (2017) and Serletis and Xu (2017) address the estimation of singular demand systems with heteroskedastic disturbances. They relax the homoskedasticity assumption implied by (8) and instead assume

$$\boldsymbol{\varepsilon}_t | I_{t-1} \sim N\left(\mathbf{0}, \boldsymbol{H}_t\right) \tag{10}$$

where $\boldsymbol{H}_t$ is measurable with respect to information set I_{t-1}. They consider a number of parameterizations for the variance model, and analytically prove the invariance of the maximum likelihood estimator with respect to the choice of the good deleted from a singular demand system. They also prove a number of important practical results regarding how to recover the mean and variance equation parameters (and their standard errors) of the full demand system from those of any subsystem obtained by deleting an arbitrary good.

In this paper, and for the first time in the interfuel substitution literature, we estimate the NQ expenditure function with heteroskedastic disturbances, following Serletis and Isakin (2017) and Serletis and Xu (2017). In doing so, we achieve superior modeling using a demand system that captures the nonlinear and time-varying features of the data. In what follows, we first discuss the data and then present the NQ expenditure function with a particular representation for the covariance matrix of the errors, in the case of three energy goods ($n = 3$).

5 The Data

In this paper we analyze aggregate primary energy consumption data, although most interfuel substitution studies in the literature analyze a single sector at a time, such as the residential or industrial sector — see, for example, Serletis *et al.* (2010) and Jadidzadeh and Serletis (2016). Given the different technological scope for fuel substitution in distinct sectors and the changing energy mix through time, the question is whether our approach in this paper is desirable. In this regard, we would like to note that data limitations make it impossible to use our model for different sectors over the same sample period as with the aggregate data. For example, sectoral data for the United States are only available (form the Energy Information Administration) from 1978 to 2011 (a total of 34 annual observations). This is also reflected in Serletis *et al.* (2010) who use U.S. data for the industrial, residential, electricity generation, and transportation sectors from 1980 to 2006 (a total of only 27 annual observations). We believe that the elasticities reported in this paper, although they are based on highly aggregated data, could be useful to policymakers contemplating policy interventions in different sectors of the economy.

We investigate interfuel substitution (energy elasticities) in the United States, using price and quantity data for the three major energy goods — crude oil (o), natural gas (g), and coal (c). In doing so, we use annual data from the U.S. Energy Information Administration (EIA) and other sources, over the period from 1919 to 2012 (a total of 94 observations). In particular, the quantity series prior to 1949 are from the 2009 EIA Annual Energy Review and after 1949 from the 2016 EIA Annual Energy Review. The consumption of oil represents petroleum consumption excluding biofuels, natural gas consumption excludes supplemental gaseous fuels, and coal consumption represents total coal consumption. For the oil, natural gas, and coal prices we use the Yatchew and Dimitropoulos (2018) annual series; they were obtained from Manthy (1978) prior to 1973 and augmented using prices published by the U.S. Energy Information Administration from 1973 to 2012. The coal prices are free-on-board prices of Anthracite coal at the point of first sale excluding freight or shipping and insurance costs, and natural gas prices are U.S. natural gas wellhead prices.

Since the Energy Information Administration reports the prices of the energy goods in different units (in particular, the price of coal, natural gas, and crude oil is expressed in dollars per metric ton, dollars per thousand cubic feet, and dollars per barrel, respectively), we convert the prices into cents per million Btu, using the EIA conversion rates as follows

1 short ton (2,000 pounds) of coal = 19,420,000 Btu
1 barrel (42 gallons) of crude oil = 5,867,946 Btu (for U.S. produced crude oil)
1 cubic foot of natural gas = 1,029 Btu.

Also, under the assumption of a representative consumer, all three quantity series were divided by population to give per capita amounts. The prices are nominal prices (cents in million Btu), because price deflators for the three energy goods are not available.

In recent years, crude oil, natural gas, and coal represent about 81% of total energy consumption in the United States. However, their relative contribution has been changing over time, as can be seen in Figure 1 which shows the consumption of these three energy goods over the sample period. On the other hand, as can be seen in Figure 2, the prices of oil and natural gas generally moved closely together until 2006 when the price of natural gas started to fall while that of oil continued to increase. Also, until the 1970s, the prices of oil and natural gas were below the price of coal.

6 The NQ with BEKK Conditional Correlations

We follow Serletis and Isakin (2017) and Serletis and Xu (2017) and use (a restricted version of) the Baba, Engle, Kraft, and Kroner (BEKK) GARCH(p, q) representation for the covariance matrix with generality parameter K [see Engle and Kroner (1995)]. In particular, we assume a BEKK GARCH$(1, 1)$ with $K = 1$ representation for the covariance matrix of the errors as follows

$$\boldsymbol{H}_t = \boldsymbol{C}'\boldsymbol{C} + \boldsymbol{B}'\boldsymbol{H}_{t-1}\boldsymbol{B} + \boldsymbol{A}'\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}'_{t-1}\boldsymbol{A}$$

where $\boldsymbol{A}$ captures the relationship between conditional variances and past residual terms, $\boldsymbol{B}$ indicates how current conditional variances and past conditional variances are correlated, and $\boldsymbol{C}$ is an upper triangular matrix containing the constant parameters in the conditional variance matrix. This specification allows past volatilities $\boldsymbol{H}_{t-1}$ and lagged values of $\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}'_{t-1}$ to show up in estimating current volatilities of each of the energy inputs.

In our case with three energy goods ($n = 3$), the NQ demand system with a BEKK specification for the covariance matrix consists of any two conditional mean equations of system (6) and the corresponding conditional variance and covariance equations. We arbitrarily delete the third energy good (coal), or, equivalently, the third share equation in (6), and thus estimate (using the maximum likelihood (ML) method) the system consisting of the following two conditional mean equations

$$s_1(\boldsymbol{v}) = \theta_1 v_1 + \frac{b_1 + \frac{\left(\sum_{j=1}^{3}\beta_{1j}v_1\right)}{\left(\sum_{i=1}^{3}\alpha_i v_i\right)} - \frac{1}{2}\frac{\left(\alpha_1\sum_{k=1}^{3}\sum_{j=1}^{3}\beta_{kj}v_k v_j\right)}{\left(\sum_{i=1}^{3}\alpha_i v_i\right)^2}}{\sum_{i=1}^{3} b_i v_i + \frac{1}{2}\frac{\left(\sum_{i=1}^{3}\sum_{j=1}^{3}\beta_{ij}v_i v_j\right)}{\left(\sum_{i=1}^{3}\alpha_i v_i\right)}} \times \left(1 - \sum_{i=1}^{3}\theta_i v_i\right) v_1 + e_1 \tag{11}$$

$$s_2(\boldsymbol{v}) = \theta_2 v_2 + \frac{b_2 + \frac{\left(\sum_{j=1}^{3}\beta_{2j}v_2\right)}{\left(\sum_{i=1}^{3}\alpha_i v_i\right)} - \frac{1}{2}\frac{\left(\alpha_2\sum_{k=1}^{3}\sum_{j=1}^{3}\beta_{kj}v_k v_j\right)}{\left(\sum_{i=1}^{3}\alpha_i v_i\right)^2}}{\sum_{i=1}^{3} b_i v_i + \frac{1}{2}\frac{\left(\sum_{i=1}^{3}\sum_{j=1}^{3}\beta_{ij}v_i v_j\right)}{\left(\sum_{i=1}^{3}\alpha_i v_i\right)}} \times \left(1 - \sum_{i=1}^{3}\theta_i v_i\right) v_2 + e_2 \tag{12}$$

and the following three conditional variance and covariance equations

$$\begin{aligned} h_{11,t} &= c_{11}^2 + b_{11}^2 h_{11,t-1} + 2b_{11}b_{21}h_{12,t-1} + b_{21}^2 h_{22,t-1} \\ &\quad + a_{11}^2 e_{1,t-1}^2 + 2a_{11}a_{21}e_{1,t-1}e_{2,t-1} + a_{21}^2 e_{2,t-1}^2 \end{aligned} \tag{13}$$

$$\begin{aligned} h_{12,t} &= c_{11}c_{12} + b_{11}b_{12}h_{11,t-1} + \left(b_{11}b_{22} + b_{12}b_{21}\right) h_{12,t-1} + b_{21}b_{22}h_{22,t-1} \\ &\quad + a_{11}a_{12}e_{1,t-1}^2 + \left(a_{11}a_{22} + a_{12}a_{21}\right) e_{1,t-1}e_{2,t-1} + a_{21}a_{22}e_{2,t-1}^2 \end{aligned} \tag{14}$$

$$\begin{aligned} h_{22,t} &= c_{12}^2 + c_{22}^2 + b_{12}^2 h_{11,t-1} + 2b_{12}b_{22}h_{12,t-1} + b_{22}^2 h_{22,t-1} \\ &\quad + a_{12}^2 e_{1,t-1}^2 + 2a_{12}a_{22}e_{1,t-1}e_{2,t-1} + a_{22}^2 e_{2,t-1}^2. \end{aligned} \tag{15}$$

7 Empirical Evidence

7.1 Theoretical Regularity

In the first column of Table 2, we report a summary of (the 'Full sample') results from the NQ demand system in terms of ML parameter estimates (with p-values in parentheses) for the mean equations (11) and (12) and the variance equations (13)-(15). We also report positivity, monotonicity, and curvature violations, as well as log likelihood values, when the model is estimated without the curvature conditions imposed. The invariance of the maximum likelihood estimates reported in Table 2 holds, irrespective of which mean equation is deleted, and also irrespective of the ordering of the remaining two mean equations.

The theoretical regularity conditions of positivity, monotonicity, and concavity are checked as in Feng and Serletis (2008), Serletis *et al.* (2010), and Jadidzadeh and Serletis (2016), as follows:

- Positivity is checked by verifying that the estimated shares are positive

- Monotonicity is checked by direct computation of the values of the first gradient vector of the estimated expenditure function
- Concavity is checked by examining whether the Slutsky matrix derived from the expenditure function is negative semidefinite.

As can be seen the unrestricted NQ model satisfies the theoretical regularity conditions at all sample observations. It is to be noted that in cases that the theoretical regularity conditions are not satisfied, one can follow Diewert and Wales (1988), and impose global concavity by setting $\boldsymbol{B} = -\boldsymbol{K}\boldsymbol{K}'$, where $\boldsymbol{K} = [k_{ij}]$ is a lower triangular matrix. See, for example, Serletis *et al.* (2010) and Jadidzadeh and Serletis (2016).

7.2 Econometric Regularity

In order to address econometric regularity issues, we conduct unit root tests in the residuals of each share equation and report the results in Table 3 (see the 'Full sample' entries). In particular, we use the Augmented Dickey-Fuller (ADF) test [see Dickey and Fuller (1981)] and the Dickey-Fuller GLS test [see Elliot *et al.* (1996)] to determine whether the share equation errors have a unit root. In doing so, we use the Bayesian information criterion (BIC) to select the optimal lag length.

In conducting the unit root tests, we first estimate the models with both intercept and trend, and find that the errors are nonstationary. However, the trend is not statistically significant and we drop it, keeping only the intercept and testing again the unit root null hypothesis. Again, we find that the errors are nonstationary, but the intercept term is insignificant, suggesting that we should conduct the unit root tests with no intercept and no trend. In this case, as shown in Table 3, the null hypothesis of a unit root can be rejected at conventional significance levels by both the ADF and DF-GLS test statistics for all three share equation residuals. We thus conclude that econometric regularity is also satisfied.

7.3 Interfuel Substitution

In the demand systems approach to the estimation of economic relationships, the primary interest, especially in policy analysis, is in the elasticity measures. Once the demand system is estimated, we can calculate different elasticity measures from the Marshallian demand functions, $x_i(\boldsymbol{v})$, $i = 1, ..., n$, in order to conduct empirical demand analysis — see Barnett and Serletis (2008) for more details.

In particular, the own- and cross-price elasticities, η_{ij}, can be calculated as

$$\eta_{ij} = \frac{\partial x_i}{\partial v_j}\frac{v_j}{x_i}, \quad i, j = 1, ..., n.$$

Also, using the homogeneity of degree zero in $(\boldsymbol{p}, y)$ property of the Marshallian demand functions, it is possible to calculate the expenditure (income) elasticities as

$$\eta_{iy} = -\sum_{j=1}^{n} \eta_{ij}, \quad i = 1, \ldots, n.$$

In addition, we can use the Allen-Uzawa and Morishima elasticities of substitution to investigate substitutability/complementarity relationships among the energy goods. The Allen-Uzawa elasticity of substitution, σ_{ij}^{a}, can be calculated as

$$\sigma_{ij}^{a} = \eta_{iy} + \frac{\eta_{ij}}{v_j x_j}.$$

However, the Allen-Uzawa elasticity of substitution may be uninformative in the case with more than two goods. In particular, for two goods the relationship is that of substitutability, but when there are more than two goods, the relationship becomes complex and depends on things such as the direction taken toward the point of approximation — see Blackorby *et al.* (1989) for more details. In that case the Morishima elasticity of substitution is the correct measure of substitution. The Morishima elasticity of substitution, σ_{ij}^{m}, can be calculated as

$$\sigma_{ij}^{m} = v_i x_i \left(\sigma_{ji}^{a} - \sigma_{ii}^{a}\right)$$

and looks at the impact on the ratio of two goods, x_i/x_j. Goods will be Morishima complements (substitutes) if an increase in the price of j, p_j, causes x_i/x_j to decrease, $\sigma_{ij}^{m} < 0$ (increase, $\sigma_{ij}^{m} > 0$).

We report the income and the own- and cross-price elasticities in Table 4, the Allen elasticities of substitution in Table 5, and the Morishima elasticities of substitution in Table 6; see the 'Full sample' entries. We do so for the (curvature unrestricted) NQ model with a BEKK specification for the conditional variance matrix, since this model satisfies full theoretical and econometric regularity. All elasticities are evaluated at the mean of the data and p-values (based on the Delta method) are given in parentheses.

As expected, all the (full sample) income elasticities reported in Table 4, η_{Oil}, η_{Gas}, and η_{Coal}, are positive ($\eta_{Oil} = 1.0257$ with a p-value of 0.0000, $\eta_{Gas} = 0.8375$ with a p-value of 0.000, and $\eta_{Coal} = 1.0686$ with a p-value of 0.000), implying that crude oil, natural gas, and coal are all normal goods, which is consistent with economic theory and the existing literature. The own-price elasticities, η_{ii}, are all negative, as predicted by the theory, and statistically significant ($\eta_{Oil,Oil} = -0.5742$ with a p-value of 0.0000, $\eta_{Gas,Gas} = -0.1431$ with a p-value of 0.000, and $\eta_{Coal,Coal} = -0.3967$ with a p-value of 0.000). For the cross-price elasticities, η_{ij}, economic theory does not predict any signs, and we note that all of the six off-diagonal terms in Table 4 are negative, indicating that the energy goods are gross complements.

In addition to the standard Marshallian income and price elasticities, we show estimates of the Allen elasticities of substitution in Table 5 (see the 'Full sample' entries). As expected, the three diagonal terms of the Allen own elasticities of substitution for the three goods are negative and highly significant ($\sigma^a_{Oil,Oil} = -1.1043$ with a p-value of 0.0000, $\sigma^a_{Gas,Gas} = -1.0840$ with a p-value of 0.0000, and $\sigma^a_{Coal,Coal} = -1.1910$ with a p-value of 0.0000), as predicted by the theory. However, because the Allen elasticity of substitution produces ambiguous results off diagonal, we use the Morishima elasticity of substitution to investigate the substitutability/complementarity relationship between the energy goods—see Blackorby *et al.* (1989) for more details.

Turning to the Morishima elasticities of substitution, it is evident from Table 6 that all the (full sample) Morishima elasticities of substitution are positive, suggesting substitutability among the energy goods in the United States. Crude oil and natural gas are substitutes (albeit not strong) to each other as indicated by the statistically significant values of the Morishima elasticities of substitution: $\sigma^m_{Oil,Gas} = 0.0443$ (with a p-value of 0.0376) and $\sigma^m_{Gas,Oil} = 0.1853$ (with a p-value of 0.0035). As already noted 4, $\sigma^m_{Oil,Gas} = 0.0443$ implies that a unit increase in the price of natural gas will increase the relative demand for crude oil by 0.0443 units; similarly, $\sigma^m_{Gas,Oil} = 0.1853$ implies that if the price of crude oil increases by one unit, the relative demand for natural gas will increase by 0.1853 units. On the other hand, the Morishima elasticities of substitution between natural gas and coal ($\sigma^m_{Gas,Coal} = 0.0912$, with a p-value of 0.0324, and $\sigma^m_{Coal,Gas} = 0.0219$, with a p-value of 0.2119) indicate that the relative demand for natural gas increases when the price of coal increases, while the reverse is not true as indicated by the statistically insignificant value for $\sigma^m_{Coal,Gas}$. Finally, there is no statistically significant substitution between crude oil and coal ($\sigma^m_{Oil,Coal} = 0.0440$, with a p-value of 0.1553, and $\sigma^m_{Coal,Oil} = 0.0235$, with a p-value of 0.5357).

In Figures 3-4 we graph the Morishima elasticities of substitution between crude oil and natural gas ($\sigma^m_{Oil,Gas}$ and $\sigma^m_{Gas,Oil}$), crude oil and coal ($\sigma^m_{Oil,Coal}$ and $\sigma^m_{Coal,Oil}$), and natural gas and coal ($\sigma^m_{Gas,Coal}$ and $\sigma^m_{Coal,Gas}$), respectively, at every point in the full sample. For these results, the Morishima elasticities of substitution between crude oil and natural gas in Figure 3 are always positive, irrespective of whether the price of crude oil or that of natural gas changes. Moreover, the Morishima elasticity of substitution between crude oil and natural gas when the price of natural gas changes, $\sigma^m_{Oil,Gas}$ (shown on the y_1 axis of Figure 3), varies between 0.02 and 0.1, whereas the Morishima elasticity of substitution between crude oil and natural gas when the price of crude oil changes, $\sigma^m_{Gas,Oil}$ (shown on the y_2 axis of Figure 3), varies between 0.10 and 0.28. The Morishima elasticities of substitution between crude oil and coal in Figure 4, confirm the general finding in Table 6 of insignificant substitution between the two energy goods. $\sigma^m_{Oil,Coal}$, shown on the y_1 axis of Figure 4, varies between 0.01 and 0.07, and $\sigma^m_{Coal,Oil}$, shown on the y_2 axis of Figure 4, varies between 0.008 and 0.06. Finally, it is apparent from the positive numbers for $\sigma^m_{Gas,Coal}$ and $\sigma^m_{Coal,Gas}$ in Figure 5 that natural gas and coal are always substitutes, but that the values are not very large. In particular, $\sigma^m_{Gas,Coal}$ (the Morishima elasticity of substitution between natural gas and coal when the price of coal changes), shown on the y_1 axis of Figure 5, varies between 0.02 and 0.14, whereas $\sigma^m_{Coal,Gas}$ (the Morishima elasticity of substitution between natural gas and coal when the price of natural gas changes), shown on the y_2 axis of Figure 5, varies between 0.007 and 0.05.

8 Subsample Analysis

We have analyzed aggregate primary energy consumption data, assuming that the data have been governed by a single process over the course of 100 years, although the crude oil, natural gas, and coal markets have undergone very significant structural changes over the past 100 years due to technological innovation, revisions in regulatory regimes and market structures, and changes in the sources of supply and demand. In this regard, Dvir and Rogoff (2010, 2014) also argue that there is a structural break in the oil price series in 1973.

To account for the structural break in the long series used in this paper and enhance the robustness of our results, we estimate the model over two subsamples, from 1919 to 1972 and from 1973 to 2012. We present the results in the corresponding columns/rows of Tables 2-6, in the same fashion as those for the full sample. As can be seen in Table 6, all the Morishima elasticities of substitution are positive and mostly statistically significant in the first subsample (from 1919 to 1972), suggesting substitutability among the different fuels. The Morishima elasticities of substitution range from 0.0115 for natural gas when the price of oil changes to 1.3197 for oil when the price of coal changes. For the 1973 to 2012 subsample, there is also evidence of substitutability between oil and coal and natural gas and coal in response to changes in the price of oil and natural gas, respectively. The evidence suggests that the demand for oil has become non-responsive to changes in the price of natural gas and coal after 1973, and that the demand for natural gas has also become less non-responsive to changes in both the price of oil and coal. The demand for coal is still being influenced by changes in the price of oil and natural gas, albeit by a smaller magnitude relative to the 1919 to 1972 subsample.

We would like to note, however, that according to the unit root test results (reported in Table 3) for the residuals of each share equation over each of the subsamples, we cannot reject the null hypothesis of a unit root. Thus, although we have achieved economic regularity in the subsample estimation, we have not achieved econometric regularity, raising questions about the validity of the subsample estimates. We believe that the elasticities for the full sample could be useful to policymakers contemplating policy interventions in the economy.

9 Policy Implications

From a policy intervention perspective, our estimated elasticities full sample could be used to investigate how the demand for energy goods will change with economic fluctuations and how taxes and subsidies will affect the level of economic activity. In particular, since coal is not responsive to crude oil or natural gas price changes, it is not possible to switch from coal consumption to crude oil or natural gas through price interventions in the crude oil or natural gas markets. However, according to our estimates, the demand for natural gas increases as the price of coal increases ($\sigma^m_{Gas,Coal} = 0.0912$), suggesting that activist price intervention in the coal market will affect the demand for natural gas. Such a policy will also have environmental implications as coal emits almost twice as much carbon than natural gas (228.6 pounds/million Btu versus 117 pounds/million Btu — see Energy Information Administration). Finally, activist and aggressive price intervention in the crude oil market will also point the way to low-carbon natural gas, but the responses will be very weak as indicated by the small Morishima elasticity of substitution ($\sigma^m_{Gas,Oil} = 0.1853$).

Overall, our results suggest that at the aggregate level, the elasticity of substitution among fossil fuels in the United States is very low, with even a potential decline in the degree of substitutability after the structural break in the oil price in the early 1970s. However, fossil fuels are very responsive to their own prices as indicated by the own price elasticities, ranging from -1.08 for natural gas to -1.19 for coal. Thus, the United States could reduce fossil fuel consumption if, for example, the consumption of these fuels was subjected to higher tax rates.

10 Comparison with Other Studies

It is difficult to provide a comparison between our results and those obtained in previous studies, mainly because, as we mentioned in the Introduction, the major contributions in this area use data before the 1970s. Also, as can be seen in Table 1, most of the early interfuel substitution studies ignore theoretical and econometric regularity issues, and employ the translog flexible functional form which fails to meet the theoretical regularity conditions of neoclassical microeconomic theory.

Regarding theoretical regularity, as Barnett (2002, pp. 199) puts it, without satisfaction of theoretical regularity, " ... the second-order conditions for optimizing behavior fail, and duality theory fails. The resulting first-order conditions, demand functions, and supply functions become invalid." Regarding econometric regularity, in this paper by focusing (for the first time in the literature) on recent advances in applied research and using relevant econometrics, we follow the suggestions in Aliprantis *et al.* (2007) and provide statistical inference regarding interfuel substitution that is internally consistent with the data and the relevant economic and econometric theory, consistent with the Lucas critique (which suggests greater integration of economic theory with econometrics).

The results are generally consistent with those reported by Serletis *et al.* (2010) and Jadidzadeh and Serletis (2016) who investigate interfuel substitution issues using the NQ functional form, treating curvature as the maintained hypothesis, as we do in this paper.

Despite the fact that the general conclusion remains the same, our estimates of the elasticities of substitution among the fossil fuels are different from those reported by Serletis *et al.* (2010) for a number of reasons. First, Serletis *et al.* (2010) use annual data from 1980 to 2006 to investigate interfuel substitution in different sectors in the United States and a number of OECD and non-OECD countries for four inputs — oil, natural gas, coal, and electricity — whereas we use aggregate annual data from 1919 to 2012 to estimate elasticities of substitution among oil, natural gas, and coal. Our methodology is also different, as we use the normalized quadratic expenditure function and relax the assumption of homoskedasticity imposing a GARCH-BEKK (1,1) specification to the conditional variance matrix. We believe that in this paper, with less restrictive econometric modelling and a larger dataset, we provide more accurate elasticity estimates.

Overall, the evidence highlights the fact that the substitution between different energy goods is quite restricted, suggesting that fossil fuels will continue to maintain their major role as a source of energy in the near future. As Serletis *et al.* (2010, p. 27) put it, "such daunting tasks as curbing carbon emissions and preventing climate change require a more active and focused energy policy. Also, because interfuel substitution is limited in the near term, there will be a greater need for relative price changes to induce switching to a lower carbon economy."

11 Conclusion

We investigate the demand for energy and interfuel substitution in the United States using (to our knowledge) the longest span prices and quantities that have ever been studied before, from 1919 to 2012. In the context of the normalized quadratic (NQ) expenditure function, we merge the empirical energy demand systems literature with recent advances in financial econometrics, relaxing the homoskedasticity assumption and instead assuming that the covariance matrix of the errors of the flexible demand system is time-varying. We also pay explicit attention to theoretical and econometric regularity.

We contribute to the literature on interfuel substitution using data going back a century, consistent with Hamilton's (1983) argument that the entire post-war period be treated as a continuous whole. We generate inference, in terms of a full set of elasticities, consistent with neoclassical microeconomic theory and the data generating process. The evidence indicates that the Morishima elasticities of substitution among the energy goods are all positive and below unity. The estimated Morishima elasticities of substitution suggest that there is a small but statistically significant substitution possibility between crude oil and natural gas. Also, natural gas is a substitute for coal when the price of coal changes, but coal is not a substitute for natural gas when the price of natural gas changes. Finally, there is no statistically significant substitutability between crude oil and coal.

Overall, our results have important implications for climate policy intervention designed to reduce greenhouse gas emissions and other pollutants generated by fossil fuel consumption. For example, they suggest limited ability to substitute coal for other energy goods than previously believed based on earlier studies. The Morishima elasticities of substitution indicate that policies that change the price of oil and natural gas do not have any impact on the demand for coal, but policies that change the price of coal will have a small but significant effect on the demand for natural gas which is a lower carbon emitting fuel.

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Table 1. A summary of flexible functional forms interfuel substitution studies

Study	Model used	Data	Goods	Curvature imposed
Berndt and Wood (1975)	Translog	Annual U.S. – Time series data – Industrial sector (1947-1971)	Labour, capital, materials, and aggregate energy	No Checked it, and is satisfied at all points
Fuss(1977)	Translog	Annual five regions of Canada – Pool of time series and cross-section – Industrial sector (1961-71)	Labour, capital, materials, and energy (coal, LPG, fuel oil, natural gas, electricity, and motor gasoline)	No
Halvorsen (1977)	Translog	Annual U.S. – Cross-section data – Industrial sector (1971)	Coal, natural gas, electricity, and fuel oil	No Tested it, but is not satisfied at all points
Pindyck (1979)	Translog	Annual ten developed countries including Canada – Pool of time series and cross-section – Industrial sector (1959-73)	Labour, capital, materials, and energy (coal, natural gas, electricity, and fuel oil)	No
Hall (1986)	Translog	Annual seven OECD countries including Canada – Time series data – Industrial sector (1960-79)	Petroleum products, natural gas, coal, and electricity	No
Considine (1989)	Translog and Linear Logit	Annual U.S. – Time series data – Industrial sector (1970-1985)	Petroleum products (residual fuel, distillate fuel, and kerosene), natural gas, coal, and electricity	No Tested it, but is not satisfied at all points
Jones (1995)	Translog and Linear Logit	Annual U.S. – Time series data – Industrial sector (1960-1992)	Petroleum products (residual fuel, distillate fuel, and kerosene), natural gas, coal, and electricity	No Checked it, but is not satisfied at all points
Serletis *et al.* (2010)	NQ	Annual 15 countries – Time series data – Residential, Industrial, Electricity generation, and Transportation sectors (1980-2006)	Oil, natural gas, coal, and electricity	Yes Satisfied at all points
Serletis *et al.* (2011)	NQ	Annual 15 countries – Pooled data – Industrial sector (1980-2006)	Oil, natural gas, coal, and electricity	Yes Satisfied at all points
Jadidzadeh and Serletis (2016)	NQ	Canada as a whole and six of its provinces – residential, commercial, and industrial sectors (1960-2007)	Electricity, natural gas, and light fuel oil	Yes Satisfied at all points

Note: This table is reproduced and updated from Jadidzadeh and Serletis (2016).

Figure 1. Consumption of the Energy Goods (in Million Btu)

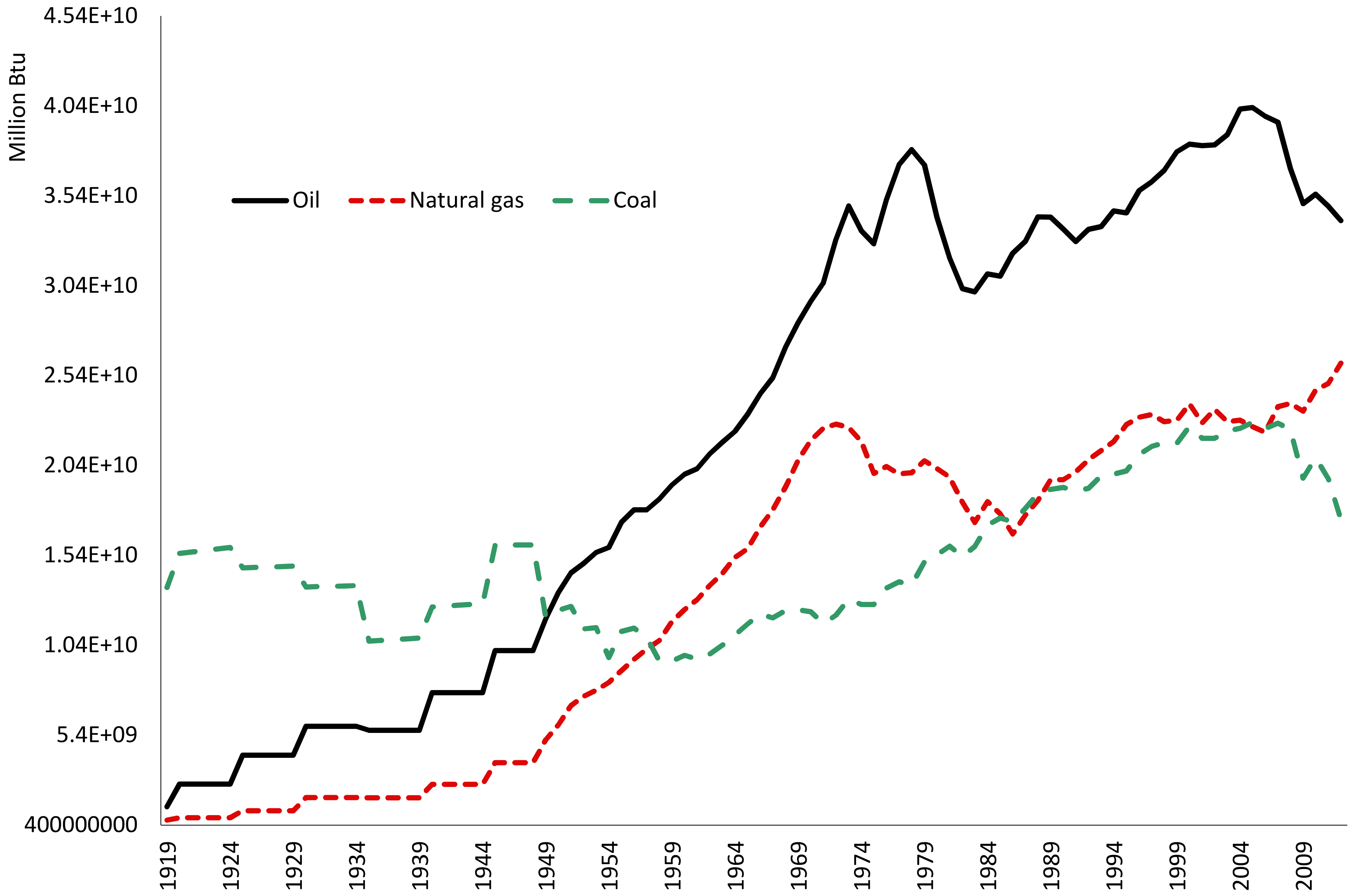

Figure 2. Prices of the Energy Goods (cents per Million Btu)

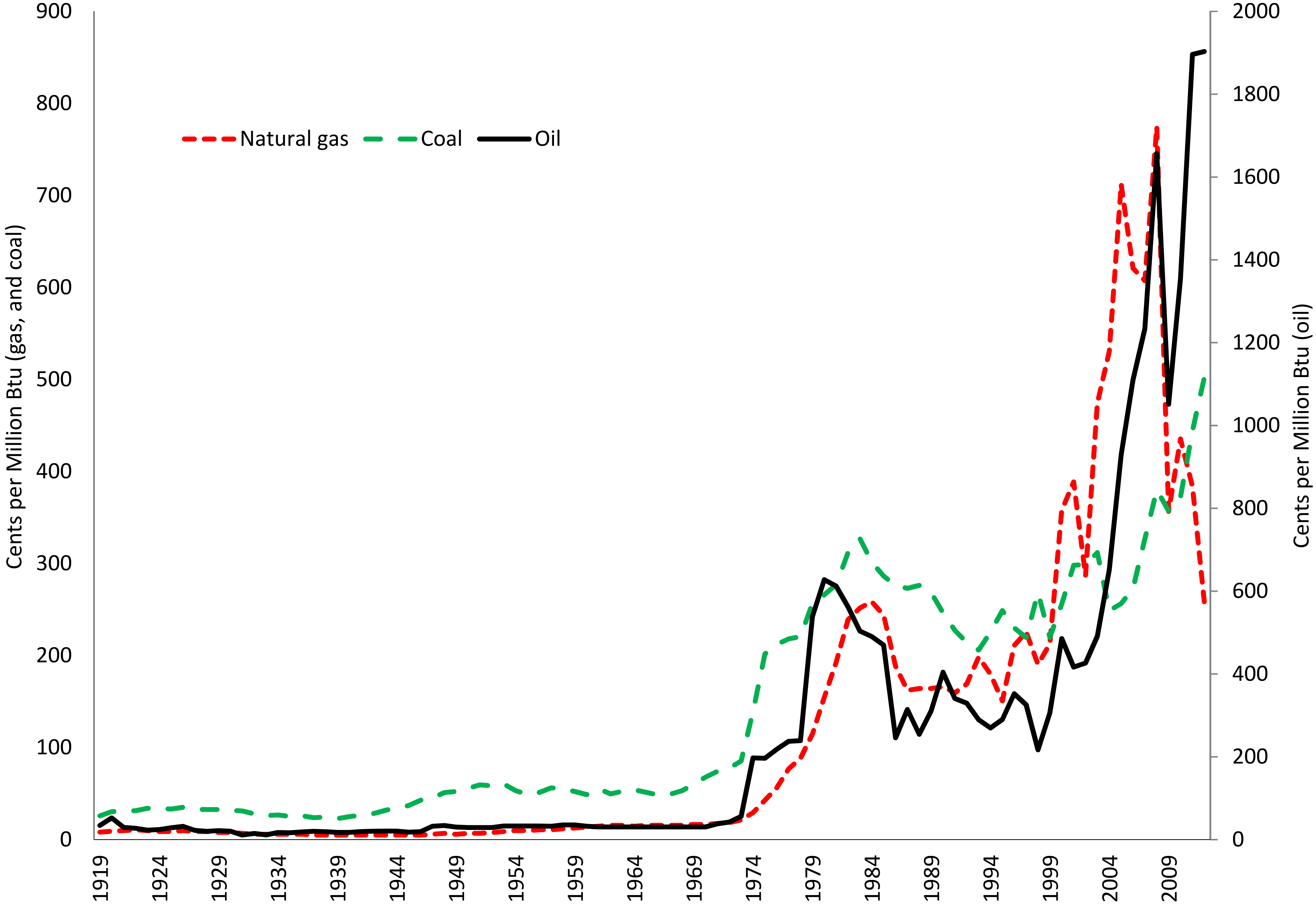

Table 2. Parameter Estimates

Goods:

1 = Crude oil
2 = Natural gas
3 = Coal

	Estimate (p-value)		
Parameter	Full sample	1919-1972	1973-2012
Mean equations			
b_1	0.6414 (.0000)	0.0590 (.0413)	0.3699 (.0000)
b_2	0.1511 (.0000)	0.0967 (.0403)	0.5314 (.0000)
b_3	0.2073 (.0000)	0.8441 (.0000)	0.0986 (.0000)
θ_1	−0.0144 (.4098)	0.0135 (.0368)	−0.0711 (.2068)
θ_2	0.0262 (.0815)	−0.0312 (.0005)	−0.0418 (.4405)
θ_3	−0.0117 (.2036)	0.0177 (.2043)	0.1130 (.0000)
β_{11}	−0.0611 (.0000)	−0.2080 (.0000)	−0.0198 (.3127)
β_{12}	0.0454 (.0000)	−0.1242 (.0002)	0.0202 (.2183)
β_{13}	0.0157 (.0000)	0.3323 (.0000)	−0.0003 (.9749)
β_{22}	−0.0406 (.0000)	−0.3024 (.0000)	−0.0419 (.0008)
β_{23}	−0.0047 (.2331)	0.4267 (.0000)	0.0217 (.0438)
β_{33}	−0.0109 (.0055)	−0.7590 (.0000)	−0.0213 (.2023)
Variance equations			
c_{11}	0.0055 (.0000)	−0.0004 (.6638)	0.0054 (.0009)
c_{21}	−0.0065 (.0000)	0.0025 (.0245)	−0.0063 (.0007)
c_{22}	−0.0000 (.9999)	−0.0000 (.9987)	−0.0000 (.9999)
a_{11}	0.9030 (.0000)	1.1824 (.0000)	1.0549 (.0000)
a_{12}	0.1561 (.0330)	0.0237 (.5801)	0.8110 (.0002)
a_{21}	0.0574 (.0030)	0.0458 (.7105)	0.0680 (.6415)
a_{22}	1.0317 (.0000)	1.1035 (.0000)	1.6862 (.0000)
b_{11}	0.3833 (.0001)	0.1252 (.0462)	−0.2933 (.0361)
b_{12}	0.0069 (.9430)	0.1780 (.0000)	0.2107 (.1514)
b_{21}	−0.0286 (.2293)	−0.3872 (.0031)	0.0792 (.0863)
b_{22}	0.3331 (.0000)	−0.5167 (.0000)	−0.0886 (.3163)
Log L	485.9933	271.0634	257.3943
Positivity violations	0	0	0
Monotonicity violations	0	0	0
Curvature violations	0	0	0

Note: Numbers in parentheses are p-values.

Table 3. Residual Unit Root Tests

Residual	Period	Test		
		ADF	ADF-GLS	Decision
Oil	Full sample	−4.928	−2.834	$I(0)$
	1919-1972	−1.106	−0.024	$I(1)$
	1973-2012	−1.355	−1.160	$I(1)$
Gas	Full sample	−2.023	−2.855	$I(0)$
	1919-1972	0.632	1.161	$I(1)$
	1973-2012	−2.206	−1.130	$I(1)$
Coal	Full sample	−5.160	−3.094	$I(0)$
	1919-1972	0.040	0.831	$I(1)$
	1973-2012	−2.540	−1.072	$I(1)$

Note: The 1% and 5% critical values for the full sample and the 1919-1972 and 1973-2012 subsamples are -2.587 and -1.943, -3.554 and -3.601, and -2.915 -2.935, respectively, for the ADF test, and -2.58 and -1.95 for the DF-GLS test.

Table 4. Income and Price Elasticities

Good i	Sample	Income	Own- and cross-price		
		η_i	$\eta_{i,Oil}$	$\eta_{i,Gas}$	$\eta_{i,Coal}$
Oil	Full sample	1.0257 (.0000)	−0.5742 (.0000)	−0.0987 (.0000)	−0.3527 (.0000)
	1919-1972	0.8963 (.0000)	−0.6562 (.0000)	−0.5290 (.0000)	0.2889 (.0339)
	1973-2012	1.2267 (.0000)	−0.4679 (.0007)	−0.6265 (.0000)	−0.1322 (.0000)
Gas	Full sample	0.8375 (.0000)	−0.3888 (.0000)	−0.1431 (.0000)	−0.3055 (.0000)
	1919-1972	1.2572 (.0000)	−0.6446 (.0000)	−0.8031 (.0000)	0.1905 (.3984)
	1973-2012	1.0817 (.0000)	−0.3814 (.0000)	−0.5991 (.0000)	−0.1011 (.0000)
Coal	Full sample	1.0686 (.0000)	−0.5506 (.0003)	−0.1212 (.0000)	−0.3967 (.0000)
	1919-1972	0.9573 (.0000)	0.0456 (.0032)	0.0278 (.2380)	−1.0308 (.0000)
	1973-2012	0.4738 (.0000)	−0.1720 (.0000)	−0.2161 (.0000)	−0.0856 (.0000)

Note: Numbers in parentheses are p-values.

Table 5. Allen Elasticities of Substitution

Good i	Period	Allen elasticities of substitution		
		$\sigma^a_{i,Oil}$	$\sigma^a_{i,Gas}$	$\sigma^a_{i,Coal}$
Oil	Full sample	-1.1043 (.0000)	-0.7478 (.0000)	-1.0588 (.0000)
	1919-1972	-5.4983 (.0000)	-5.3342 (.0000)	0.3600 (.0319)
	1973-2012	-1.5760 (.0007)	-1.2846 (.0000)	-0.5795 (.0000)
Gas	Full sample		-1.0840 (.0000)	-0.9171 (.0000)
	1919-1972		-8.0986 (.0000)	0.2374 (.3961)
	1973-2012		-1.2285 (.0000)	-0.4432 (.0000)
Coal	Full sample			-1.1910 (.0000)
	1919-1972			-1.2844 (.0000)
	1973-2012			-0.3753 (.0000)

Note: Numbers in parentheses are p-values.

Table 6. Morishima Elasticities of Substitution

Good i	Period	Morishima elasticities of substitution		
		$\sigma^m_{i,Oil}$	$\sigma^m_{i,Gas}$	$\sigma^m_{i,Coal}$
Oil	Full sample		0.0443 (.0376)	0.0440 (.1533)
	1919-1972		0.2741 (.0060)	1.3197 (.0000)
	1973-2012		-0.0273 (.8407)	-0.0465 (.1668)
Gas	Full sample	0.1853 (.0035)		0.0912 (.0324)
	1919-1972	0.0115 (.8550)		1.2214 (.0000)
	1973-2012	0.0865 (.4872)		-0.0154 (.4966)
Coal	Full sample	0.0235 (.5357)	0.0219 (.2119)	
	1919-1972	0.7018 (.0000)	0.8310 (.0000)	
	1973-2012	0.2959 (.0172)	0.3830 (.0000)	

Note: Numbers in parentheses are p-values.

Figure 3. Morishima Elasticities of Substitution Between Crude Oil and Natural Gas

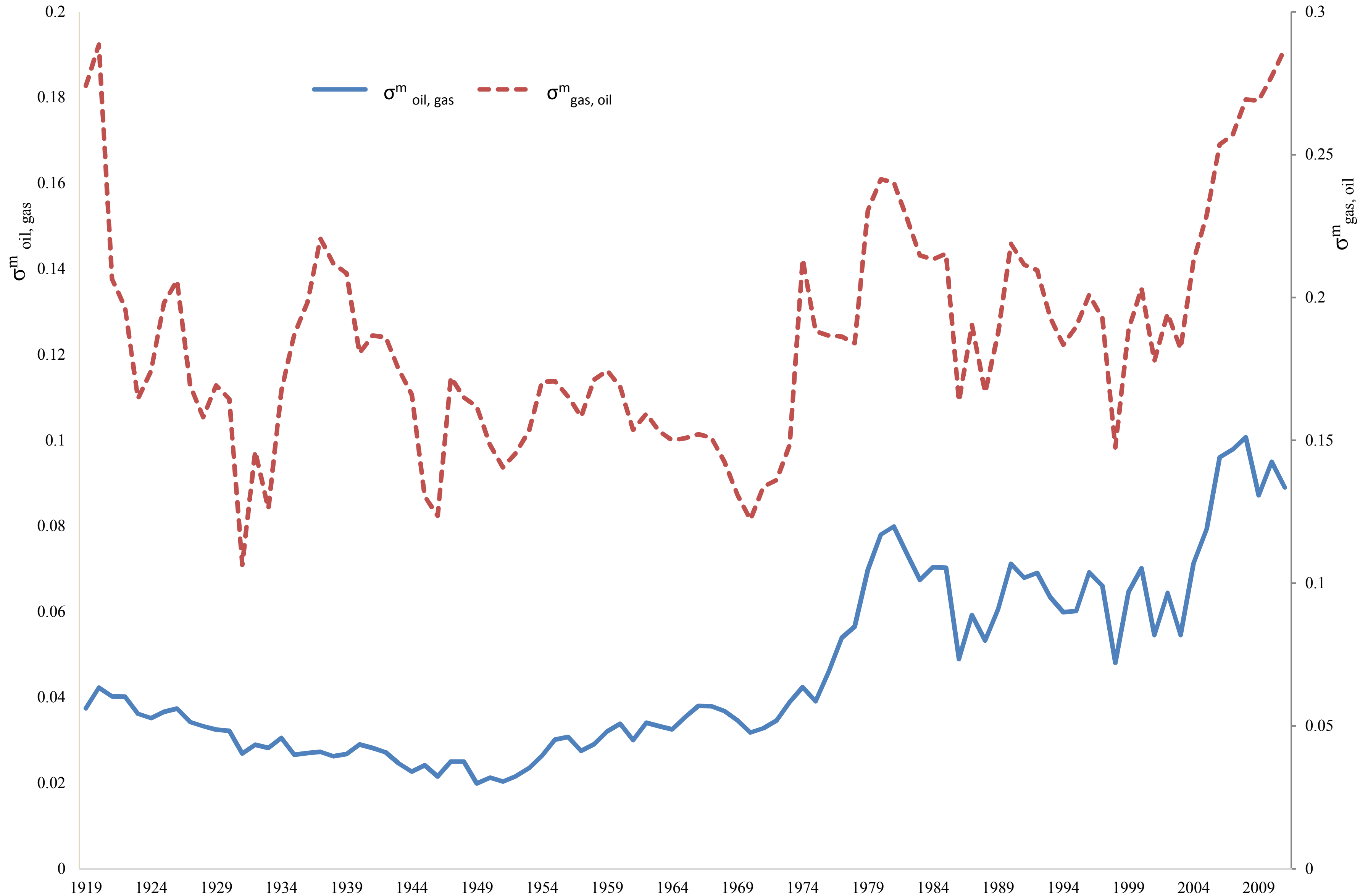

Figure 4. Morishima Elasticities of Substitution Between Crude Oil and Coal

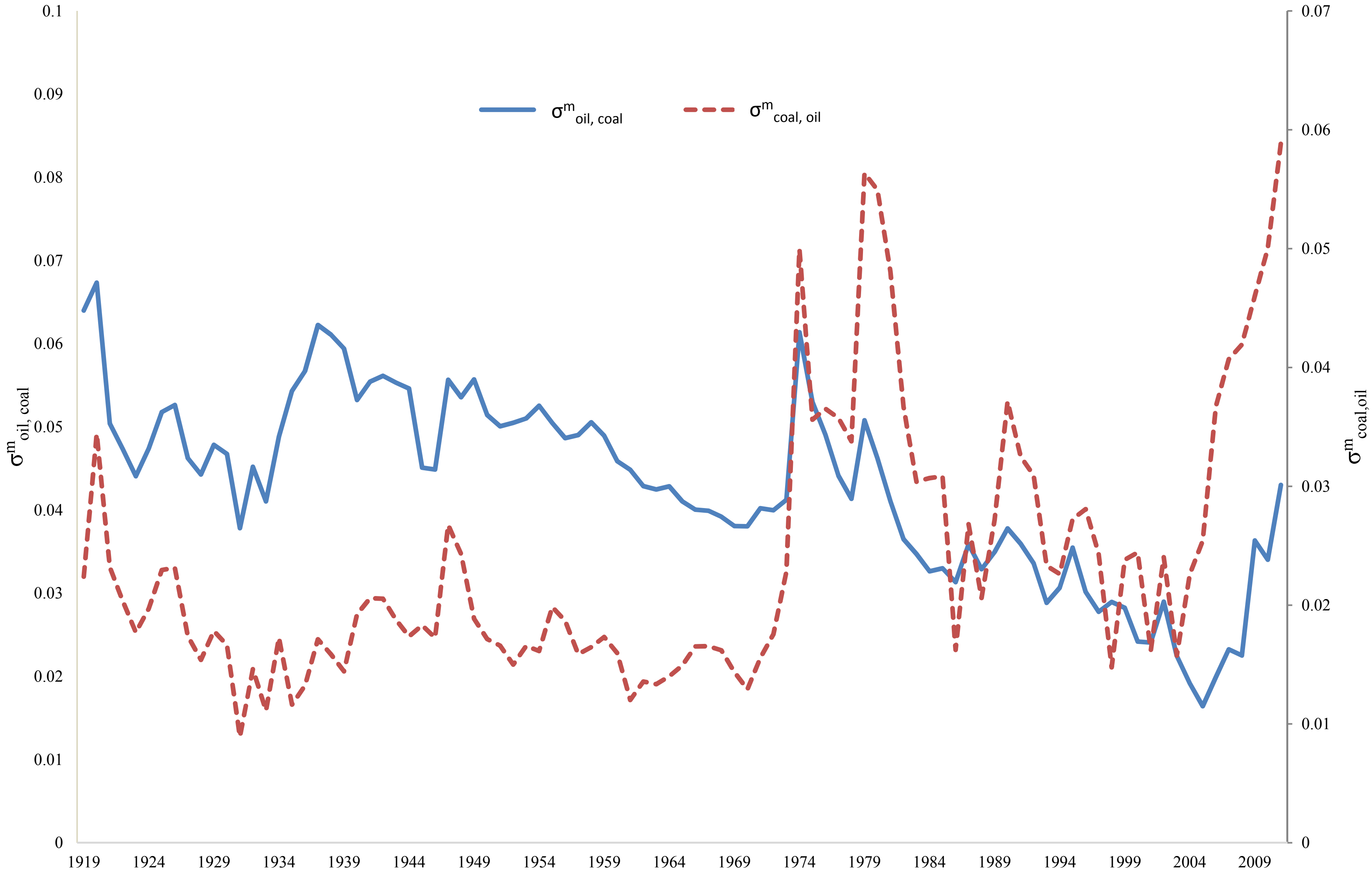

Figure 5. Morishima Elasticities of Substitution Between Natural Gas and Coal